

Non-perturbative Gluons and Pseudoscalar Mesons in Baryon Spectroscopy

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Abstract

We study baryon spectroscopy including the effects of pseudoscalar meson exchange and one gluon exchange potentials between quarks, governed by α_s . The non-perturbative, hyperspherical method calculations show that one can obtain a good description of the data by using a quark-meson coupling constant that is compatible with the measured pion-nucleon coupling constant, and a reasonably small value of α_s .

Interest in studying baryon spectroscopy has been re-vitalized by the recent work of Glozman and Riska [1–5]. These authors point out the persistent difficulty in obtaining a simultaneous description of the masses of the P-wave baryon resonances and the Roper-nucleon mass difference. In particular they argue [2] that “the spectra of the nucleons, Δ resonances and the strange hyperons are well described by the constituent quark model, if in addition to the harmonic confinement potential the quarks are assumed to interact by exchange of the $SU(3)_F$ octet of pseudoscalar mesons”. Furthermore, Ref. [5] states that gluon exchange has no relation with the spectrum of baryons !

The ideas of Glozman and Riska are especially interesting because of the good descriptions of the spectra obtained in Refs. [1]– [5], and because of the contradictory long-standing belief [6–9] that one-gluon exchange is a basic element of quantum chromodynamics QCD and the success of that interaction in baryon spectroscopy. Despite the lore, some authors had noted the difficulty in obtaining a simultaneous description of the Roper and P-wave resonances [10,11].

The purpose of this paper is to include both effects in calculating the baryon spectra using a non-perturbative technique, and to show that both kinds of effects are required for a reasonable description of the data. Including the effects of pion clouds is known to lead to a good description of nucleon properties, as well as meson-nucleon and electron-nucleon scatterings [12,13]. We note that several previous workers [14]– [17] have shown that including both pion exchange and gluon exchange effects leads to an improved description of the data. Those calculations use a perturbative treatment of the pion and gluon exchange interactions.

However, non-perturbative calculations are required to handle the one-gluon exchange interaction [11,18,19]. It is therefore natural to expect that if one used only pseudoscalar meson exchange to generate all of the mass splitting, a non-perturbative treatment would be necessary. Thus a non-perturbative, all-orders treatment is needed to assess whether or not either of those two elements can be ignored. We employ the hyperspherical methods of Fabre de la Ripelle et al [20] to compute the energies of the baryons.

We use a constituent quark model Hamiltonian which includes the effects of one gluon exchange (OGE) and the exchange of pseudoscalar mesons mandated by broken chiral symmetry, V_χ , in addition to the kinetic energy and confinement terms. Thus

$$H = T + V_{con} + V_{OGE} + V_\chi, \quad (1)$$

where the kinetic energy T takes the non-relativistic form

$$T = \sum_i -\frac{\nabla_i^2}{2m}, \quad (2)$$

with the u or d quark mass taken as 336 MeV to represent the non-perturbative effects which influence the properties of a single confined quark. We limit ourselves to light quarks in this first calculation, but note that the success in handling strange baryons in an important part of the work of Glozman and Riska.

Here we assume that the confining interaction V_{con} takes on a linear (V_L) form so that:

$$V_L = \sum_{i < j} A_L \mid \vec{r}_i - \vec{r}_j \mid. \quad (3)$$

The parameter A_L is to be determined phenomenologically. The one gluon exchange interaction between different quarks is given by the standard expression

$$V_{OGE} = \sum_{i < j} \left[-\frac{2\alpha_s}{3r_{ij}} + \frac{2\pi\alpha_s}{3m^2} \frac{1}{4\pi} \frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} - \alpha_s \frac{4}{9} \frac{\pi}{m^2} \frac{1}{4\pi} \frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} \vec{\sigma}_i \cdot \vec{\sigma}_j \right], \quad (4)$$

where $r_{ij} \equiv \mid \vec{r}_i - \vec{r}_j \mid$, $r_0 = 0.238$ fm, and α_s is a parameter to be determined phenomenologically. The replacement of the usual delta function form by a Yukawa of range r_0 is intended to include the effects of the finite sized nature of the constituent quarks.

We ignore the spin-orbit and tensor terms because our first calculation is intended to be a broad comparison of the non-perturbative effects of gluon and meson exchange. Isgur and Karl [21] found that including the tensor hyperfine forces with relative strengths predicted by the one gluon exchange interaction is necessary to produce the splitting between the $J^\pi = 1/2^-$ and $J^\pi = 3/2^-$ nucleonic states as well as to understand their separate wave functions and consequent decay properties. Therefore we do not expect our calculations

to reproduce those features. The issue of the spin-orbit interaction between quarks is a complicated one. There are many different contributions: Galilei invariant and non-invariant terms arising from one gluon exchange see e.g. [22], a Thomas precession term arising from the confining interaction [7], effects of exchange of scalar mesons and the instanton induced interaction [23]. The above cited authors show that some of the various terms tend to cancel when evaluating the baryon spectra. A detailed study of the influence of the various contributions to the spin orbit force is beyond the scope of the present work.

The effects of pseudoscalar meson octet exchange are described by the interaction [1]- [5]

$$V_\chi = \sum_{i < j} \alpha_{q\pi} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{3} \frac{\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F}{4 m^2} [\mu^2 \frac{e^{-\mu r_{ij}}}{r_{ij}} - \frac{e^{-r_{ij}/\Lambda}}{\Lambda^2 r_{ij}}], \quad (5)$$

where $\Lambda = 0.238$ fm [16] represents the effects of the finite size of the constituent quarks. We shall allow the strength of the meson exchange potential, $\alpha_{q\pi}$, to vary away from the expected [2] value of 0.67. This is in the spirit of the work of Refs. [1]- [5] who fit a very few matrix elements of V_χ to a few mass differences and predict the remainder of the spectrum. The values of the flavor SU(3) matrices are taken from Eq. (5.1) of Ref. [2]. We neglect the tensor force generated by the exchange of pseudoscalar mesons, as do Glozman and Riska. Similarly, retardation effects and the influence of the baryonic mass differences are neglected.

Next we turn to a brief description of the hyperspherical method, which has been in use for some time [20,24]. The idea is that the Schroedinger equation for three particles can be simplified by expressing the usual Jacobi coordinates $\vec{\xi}_1 = \vec{r}_1 - \vec{r}_2$ and $\vec{\xi}_2 \equiv \frac{1}{\sqrt{3}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$ using the hyperspherical coordinates defined by a radial distance $r = \sqrt{\xi_1^2 + \xi_2^2}$, polar angles $\omega_i = (\theta_i, \phi_i)$ of $\vec{\xi}_i$, and the additional angle ϕ defined as $\tan\phi = \xi_2/\xi_1$. The hyperspherical harmonics consist of a complete set of angular functions on the 5-dimensional hypersphere. Hence the wave function and potential can be expressed in terms of linear combinations of these functions. Furthermore, Ref. [25] has shown how to construct linear combinations of these functions that form irreducible representations of the permutation group of three particles in the S-state. This enables one to construct wave functions that are consistent with the Pauli exclusion principle. In particular, the requirement of constructing color-

singlet states is met by treating the wave function as a product of the standard SU(6) spin-flavor wave functions, by symmetric spatial wave functions, by the anti-symmetric color wave function. Therefore the effects of mixed symmetry states are ignored here.

The basis of hyperspherical harmonics has a large degeneracy, which can be handled by using the optimal subset [26] which is constructed as linear combinations of Potential Harmonics, i.e., those states generated by allowing the potential $V_{con} + V_{OGE} + V_\chi$ to act on the hyperspherical harmonics of minimal order allowed by the Pauli exclusion principle. See Ref. [24] for a detailed discussion of the general formalism. The convergence properties of the expansion and the accuracy of using a single optimal state have been studied by several authors [27,28] with the result that the overlap between the approximate and exact eigenfunctions is generally greater than 99.5%.

To be specific, we display the specific nucleon and Δ wave functions. The nucleon wave function is given by

$$\psi^N = \frac{1}{\sqrt{2}} \left[\chi^\rho \eta^\rho + \chi^\lambda \eta^\lambda \right] u_N(r) r^{-5/2}, \quad (6)$$

where $\chi^\rho, (\eta^\rho)$ are the mixed antisymmetric spin (flavor) wave functions and $\chi^\lambda, (\eta^\lambda)$, are the mixed-symmetric spin (flavor) wave functions. The Δ wave function is given by

$$\psi^\Delta = \chi^{3/2} \eta^{3/2} u_\Delta(r) r^{-5/2}. \quad (7)$$

The radial wave functions u_N and u_Δ are obtained by solving the differential equation:

$$\left[\frac{\hbar^2}{m} \left(-\frac{d^2}{dr^2} + \frac{15/4}{r^2} \right) + V_{N,\Delta}(r) - E \right] u_{N,\Delta}(r) = 0, \quad (8)$$

where the potentials $V_{N,\Delta}(r)$ are obtained by re-expressing the interactions above in terms of a quark-quark interaction V_{qq} such that

$$V_{qq}(r_{ij}) = V^0(r_{ij}) + V^S(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j + V^\chi(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F. \quad (9)$$

The term V^0 includes both the confining and spin independent part of the quark-quark interaction. Then the potential $V(r)$ of Eq. (8) is given by

$$\begin{aligned}
V_N(r) &= \frac{48}{\pi} \int_0^1 \left[V^0(r u) - V^S(r u) + C_N V^X(r u) \right] \sqrt{1-u^2} u^2 du, \\
V_\Delta(r) &= \frac{48}{\pi} \int_0^1 \left[V^0(r u) + V^S(r u) + C_\Delta V^X(r u) \right] \sqrt{1-u^2} u^2 du,
\end{aligned} \tag{10}$$

where $C_N = 14/3$ and $C_\Delta = 4/3$ are obtained by taking the matrix elements of the flavor-spin matrix $\vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F$ in the appropriate wave functions. The differential equations are solved using the renormalized Numerov method formulated by Johnson [29].

The first model we shall consider includes the one-gluon exchange but neglects the effects of the meson exchange interaction V_χ . The differences between the computed and measured values of the mass splitting are shown as a function of α_s in Fig.1. A curve passes through the horizontal line when the computed value of the indicated mass difference is equal to the experimental value of that difference. This notation is used in each of the figures. The results of Fig.1a show how the model can account for the splitting between the Δ and nucleon, Δ^* and the Δ , and the Roper and nucleon, but not the splitting between the P-wave resonance and the nucleon. Note that a large value of $\alpha_s \approx 2.2$ is used to obtain the fit with $A=0.10$ GeV/fm. If one uses instead $A=0.45$ GeV/fm, one is able to account for the Δ -nucleon and P-wave nucleon splitting but not the Roper, as shown in Fig. 1b. This agreement is obtained also for a large value of $\alpha_s \approx 1.4$ that roughly corresponds to the original theory of Refs. [6,7] which works reasonably well except for the Roper.

One may also study the converse situation of keeping pseudoscalar meson exchange and ignoring the gluonic exchange, which represents a non-perturbative treatment of the Glozman-Riska theory. The results, shown in Fig. 2, indicate that this version of the non-relativistic quark model is very successful if one allows the freedom to vary the value of $\alpha_{q\pi}$ away from the expected value of 0.67 [2]. Using a factor of two increase so that $\alpha_{q\pi} \approx 1.4$ improves immensely the agreement with experiment. No such agreement can be obtained if one insists on using the value 0.67. Note also that the energy of the $3/2^-$ state is not too well described.

The third model we consider is the most general, in which both the color magnetic and pseudoscalar meson exchange terms are included. Both of these terms contribute to the N- Δ

splitting [12], so that including both effects can be reasonably expected to lead to smaller values of α_s and $\alpha_{q\pi}$ than used in Figs. 1 and 2. The results for this general model are shown in Fig. 3. One obtains a good description of the data, with the energy of the state $N3/2^-$ state as the expected single exception. Furthermore, the value of α_s is about 0.7 instead of about 2 required if this is the sole physics responsible for the Δ -nucleon mass splitting. A smaller value is preferred because this interaction is derived using perturbation theory. Still another nice feature is that the value of $\alpha_{q\pi} \approx 1$ which is close the value expected from the measured pion nucleon coupling constant, $g_{\pi N}$. The relation between the pion-quark coupling constant, g , and $g_{\pi N}$ is $g_{\pi N} = \frac{m_u}{g_A m_N} g$ [2] Using the experimentally measured axial coupling constant $g_A = 1.26$ along with our quark mass $m_u = 336$ MeV and $\frac{g_{\pi N}^2}{4\pi} = 14.2$ gives $\alpha_{q\pi} = \frac{g^2}{4\pi} = 1.1$. The use of $g_A = 1.26$ accounts for known relativistic effects, which change the quark wave functions but do not modify the spectrum [8]. The use of $\alpha_{q\pi} \approx 1$ to reproduce the differences between baryon masses therefore represents a significant improvement in the theory.

We have obtained a good description of the energies of states, so that it is worthwhile to begin discussing some of the properties of the wave functions. We note that the value of $A_L = .17$ GeV/fm, which yields a nucleon rms radius of 0.46 fm is significantly smaller than the experimental value ~ 0.8 fm, but much larger than obtained, ≈ 0.3 fm, in work using only one gluon exchange such as that of Refs. [18,19]. We note that including the relativistic recoil correction, also invoked by Capstick and Isgur, is known to increase the computed value of the radius. Similar effects occur by including the influence of the meson cloud on the nucleon radius, and the effects of other components of the wave function. We plan to include such effects, along with tensor and spin orbit forces and retardation effects in future work. This would enable us to obtain a realistic treatment and to compute the decay properties of the excited states. We also plan to consider strange baryons.

The net result of the present work is that non-relativistic calculations including confinement, one gluon and pseudoscalar meson exchange can describe the light-quark baryon spectrum reasonably well. Most of the mass differences between the states are described

within accuracy of 10 % or better. In particular, we find that including the effects of meson exchange leads to a good simultaneous description of the Roper-N and P-wave-nucleon splitting even if the one gluon exchange interaction is neglected. This is in agreement with Riska and Glozman. However, both gluonic and pseudoscalar meson exchange are expected from the underlying theory. One also gets a good description of the baryon energies in this more general theory, with the improvements that the value of α_s is smaller than before and the value $\alpha_{q\pi}$ is very close to the one provided by the pion-nucleon coupling constant. Thus although we verify several of the statements of Refs. [1]- [5], a theory which includes both gluon and meson exchange seems more plausible.

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FIGURES

FIG. 1. Baryon mass splitting versus α_s , with $V^\chi = 0$. Differences between the computed and measured values of the mass splitting Δ (in GeV) are shown. a) $A=0.10$ GeV/fm b) $A=0.45$ GeV/fm.

FIG. 2. Baryon splitting- Glozman Riska model, neglecting the one gluon exchange interaction, $V_{OGE} = 0$. The mass differences (Δ in GeV) are shown as a function of $\alpha_{q\pi}$.

FIG. 3. Baryon splitting with the complete Hamiltonian. The mass differences (Δ in GeV) are shown as a function of α_s

Fig.1 a

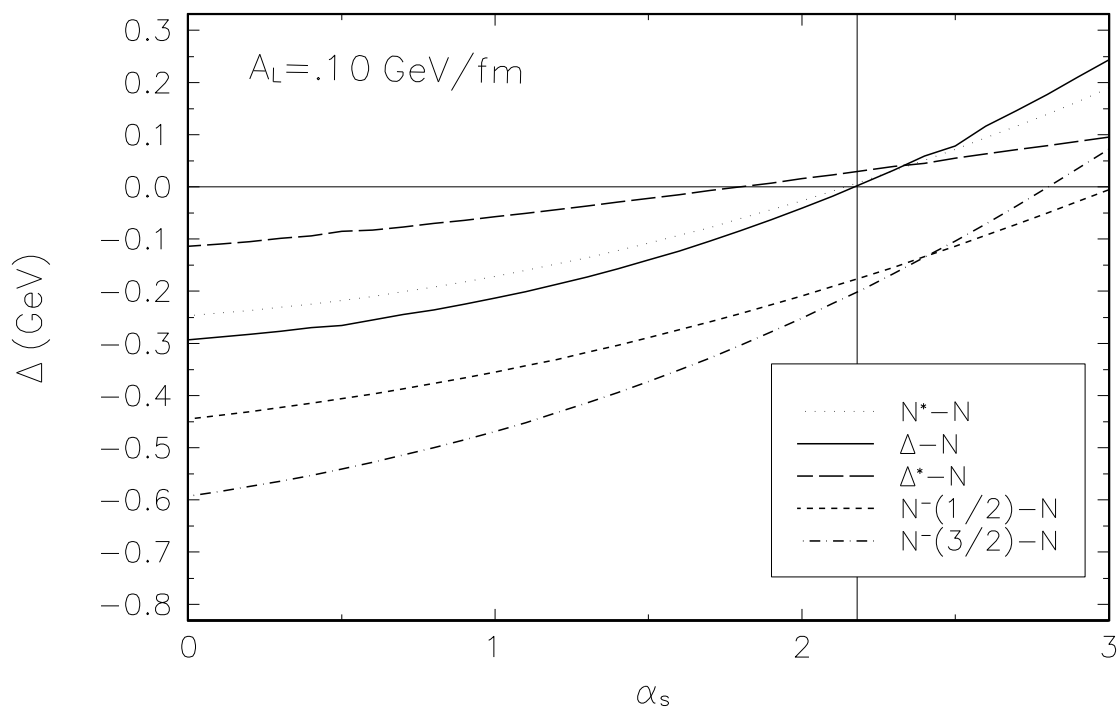


Fig.1 b

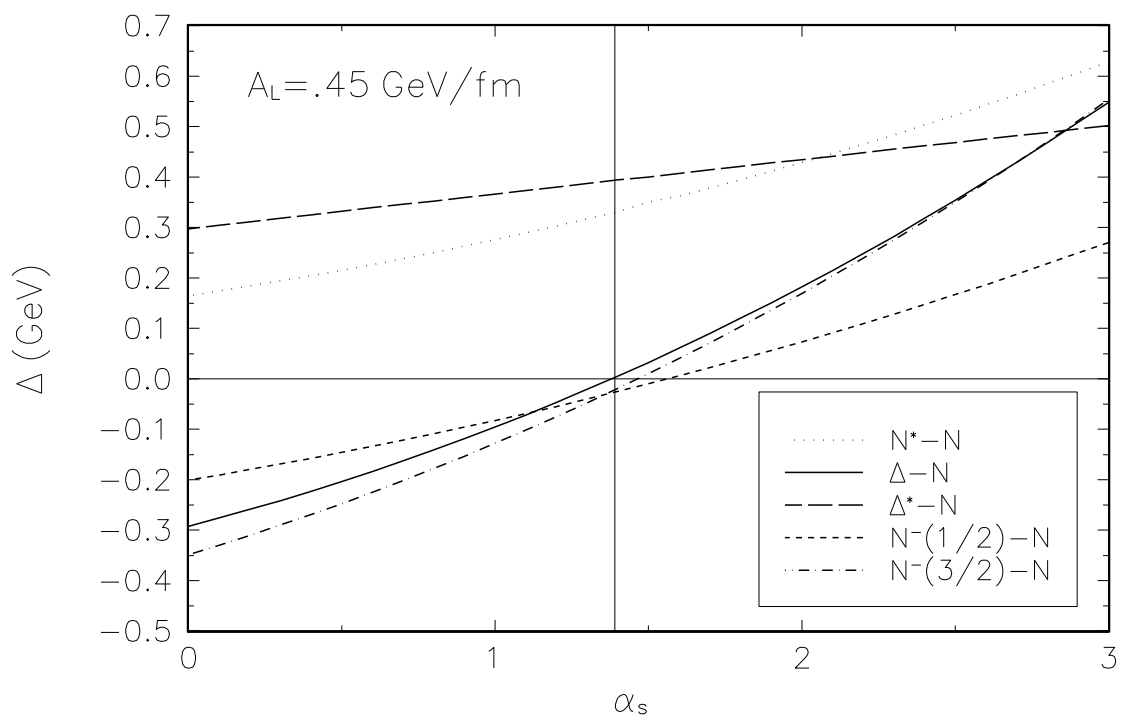


Fig.2

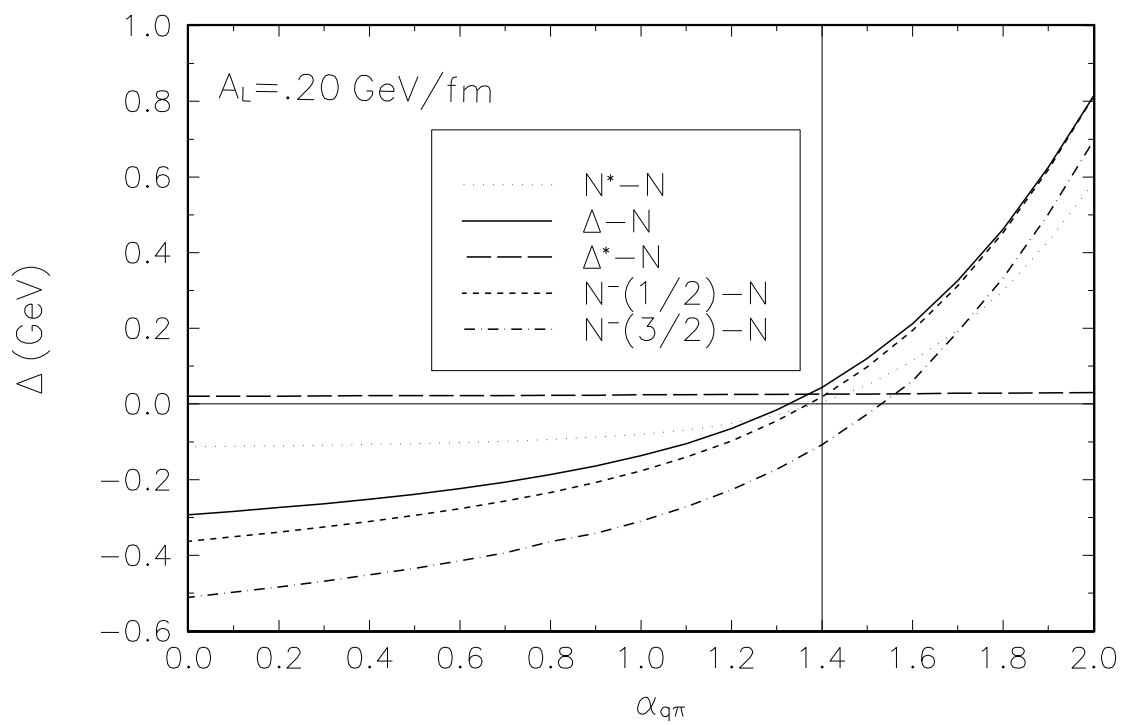


Fig.3

